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ABSTRACT

This guide describes activities and materials which can be used in a mathematics laboratory approach for a basic mathematics program for grades 1-6. Forty-seven activities, concerning number and numeration, are described by their purpose, suggested grade levels, materials needed, and procedures. Concepts presented include: counting, number recognition, sets, measurement, estimation, place value, addition, subtraction, multiplication and division facts, prime and composite numbers, mathematical vocabulary, applications, weighing, and monetary values. The booklet contains a list of manipulative materials for mathematics laboratory use, including improvised materials and games, commercial materials and games, general supplies, and storage containers. (JBW)

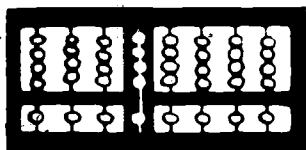
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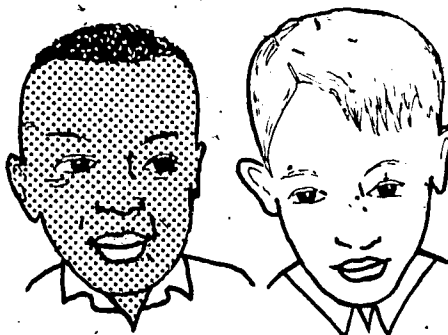
SUGGESTIONS FOR TEACHING MATHEMATICS USING LABORATORY APPROACHES GRADES 1-6

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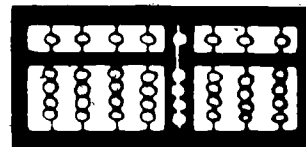


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1. NUMBER AND NUMERATION

The University of the State of New York
THE STATE EDUCATION DEPARTMENT

Bureau of Elementary Curriculum Development

Albany, New York 12224

Reprint 1974

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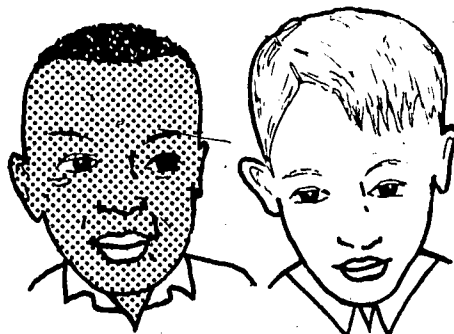
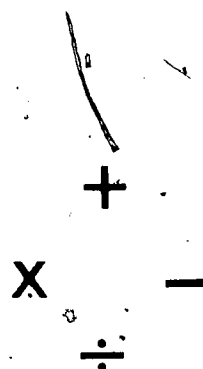
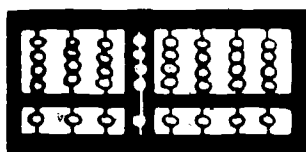
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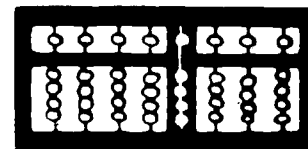
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$3 > 1$

$5 < 6$



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PREFACE

Substantial financial aid to local educational agencies for children of low income families was provided by the Elementary and Secondary Education Act of 1965. Participating school districts have developed a variety of new educational programs to assist children with special educational needs. These programs are based upon local needs assessment with major parental involvement.

The three major priorities for compensatory education programs are bilingual education, reading, and mathematics. One of the highest priority programs under ESEA Title I is the subject matter area of mathematics. Experience has shown that children who have experienced difficulty learning in a traditional program often react with enthusiasm to a mathematics laboratory approach. This publication was developed to provide practical applications of this mathematics approach for use by classroom teachers. It should provide practical suggestions for teachers working directly with educationally disadvantaged children.

Irving Ratchick
Assistant Commissioner for
Compensatory Education

FOREWORD

The Bureau of Elementary Curriculum Development and Bureau of Mathematics Education in cooperation with the Division of Education for the Disadvantaged, ESEA Title I, have developed a variety of materials on the use of a Mathematics Laboratory approach on the elementary level. This joint effort has resulted in the release of two publications:

Teaching Elementary Mathematics Using Laboratory Approaches,

which serves as a short introduction to the method

and

ESEA Title I, Anatomy of an Elementary Project, which gives a concrete example of the use of a Mathematics Laboratory approach with disadvantaged children.

Encouraged by the response of teachers and administrators to the original publications, a decision was made to move further in the direction of providing concrete activities for teachers who wished to move into the humanistic approach inherent in a Mathematics Laboratory program.

Fredric Paul of the Bureau of Mathematics Education and Peter A. Martin of the Bureau of Elementary Curriculum Development began the task of developing activities for teacher use. A committee of experienced teachers consisting of Claire Cohn, Helen Feder, and Pasquale Toscano, under the direction of Elaine Mintz, was engaged as a writing team. Mrs. Mintz is director of elementary mathematics and the other three are teachers in the Plainedge School District. The material completed by this team was then sent out for field testing to 11 schools throughout the State for use with children. As a result of a favorable reaction on the part of the teachers who used this material, we have produced

experimental materials for use by school districts.

This publication is the first of four which are being developed for teacher use. Each will incorporate the latest thinking of the mathematics revision committee and may be utilized with any basic mathematics program. This publication is designed to serve as a stimulant to encourage teachers to open their minds and employ their imaginations in developing further activities. The classroom teacher in developing her own set of "task cards" will adjust vocabulary and choose concrete materials in terms of a close knowledge of the ability levels of her own children and the type and amount of manipulative materials available. Suggestions and reactions are welcome and should be sent to Fredric Paul, Bureau of Mathematics Education, State Education Department, Albany, New York 12224.

Peter A. Martin of the Bureau of Elementary Curriculum did the final editing and prepared the material for publication.

Robert H. Johnstone, Chief
Bureau of Elementary
Curriculum Development

Gordon E. Van Hooft, Director
Division of School Supervision

1 - NUMBER AND NUMERATION

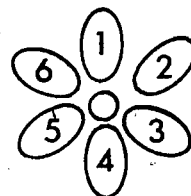
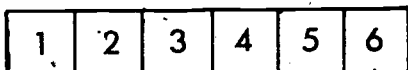
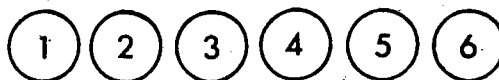
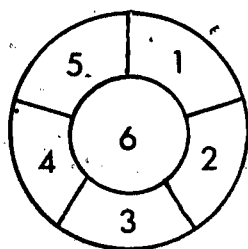
1 - NUMBER AND NUMERATION

1-1 Cover Alls. Purpose: Counting, number recognition, number line readiness

Suggested Grade Level: 1, 2

Materials needed: Dice or numbered spinners, number strips, counters

Procedure: Provide each player with a strip, or other pattern, numbered 1 through 6 and six counters. Each player takes a turn rolling a die or spinning. He covers the numeral on his strip that corresponds to the number rolled. The first player to cover all the numbers wins. The game can be extended to numbers greater than 6 if more than one die or spinner is used.



1-2 Do as I Do. Purpose: Pattern discovery, sets

Suggested Grade Level: K, 1, 2

Materials needed: Counters which can be assembled and dismantled easily, as: stringing or popit beads, unifix materials, geoboards, attribute blocks, and Cuisenaire rods

Procedure: The teacher may display patterns and ask pupils to duplicate these patterns. Patterns can involve differentiation by any distinguishing characteristic. Pupils can work in pairs to create their own patterns and duplicate each other's work.

The teacher may also hold up a card with a numeral and have the pupils use as many of the materials available as needed to represent that numeral.

1-3 Picture Line. Purpose: Cardinal and ordinal numbers, vocabulary, directionality, counting, 1 to 1 correspondence, adding, subtracting, sequence, spacial relationships, numberline readiness

Suggested Grade Level: 1, 2

Materials needed: Pictures, tape

Procedure: Collect a set of pictures (start with five about which you can elicit a sequential story.) As the story unfolds, tape these pictures in a straight line on the floor, spaced a child's step apart. Discuss what happened first, second, etc. Where would you stand on the picture line to show the beginning of the story? Establish that each picture in the line can be reached by taking one step from the start or from the picture before it.

Activities: Start at the beginning and take two steps. On which picture did you land? Walk to the picture of the _____. How many steps did it take for you to get there? How many steps is the _____ from the _____? How many steps are between _____ and _____? Which picture(s) is (are) two steps from _____? Take two steps and one more; on which picture are you? How many steps from the starting place are you? etc.

1-4 Arrangements. Purpose: Grouping, sets, subsets.

Suggested Grade Level: 1, 2, 3

Materials needed: Attribute blocks, cardboard shapes

Procedure: Arrange the set of blocks so that each subset (group) has only those pieces that have the same color and the same shape. How many subsets are there? How many blocks in each? How do the pieces within a subset differ from each other? Give each group a name.

1-5 Overlaps. Purpose: Intersection of sets

Suggested Grade Level: 1, 2, 3

Materials needed: Attribute blocks

Procedure: Choose a color. Make a subset of all pieces with the color (value). Choose a shape. Make a subset of all pieces with the shape. Now, from these subsets take all the pieces that have both the color and shape you chose. How many pieces in the first subset? in the second? How many pieces share both values (the color and the shape). Vary, using other colors, shapes, sizes.

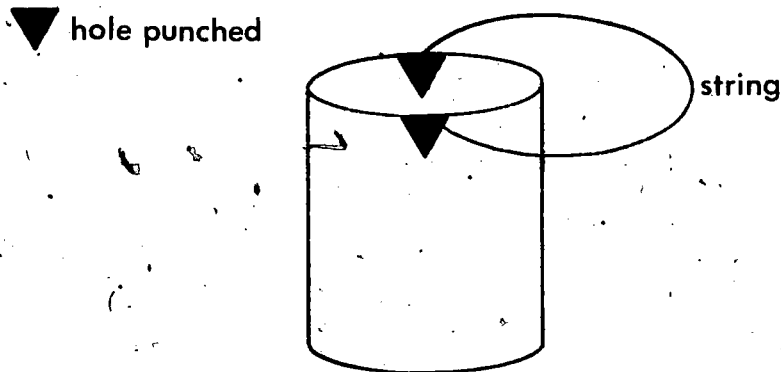
1-6 Stilts. Purpose: Measurement, rote counting, addition, estimation, graphing, larger-smaller, greater than-less than, eye-hand coordination

Suggested Grade Level: 1, 2

Materials needed: Large cylindrical cans, string

Procedure: Make stilts by using large cylindrical cans. Make a hole on each side and pull heavy string through, long enough to reach a child's hands. Knot tightly so the child can pull up on it. Have children measure each other's pace. Estimate how many paces it will take to reach the other side of the room. Estimate whether it will take more or less steps than a given number. Try it. Mark position of where to begin and where to end. Pace distance. Check it. How close was the guess? Record the findings on a graph.

Approximately 6" high, 4" diameter



1-7 What Am I ? Purpose: Set classification

Suggested Grade Level: 2, 3

Materials needed: Attribute blocks

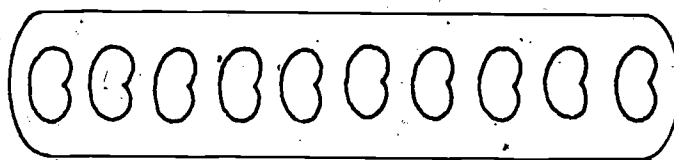
Procedure: Arrange all the red and blue circles and diamonds in an orderly way. One of the pieces is not red, not a diamond, not small. What is it? Vary, using other properties. Children may play this as a game, taking turns with a partner. Try to play without looking at the pieces.

1-8 Beanstick Place Value. Purpose: Place value, counting, addition, subtraction, expanded notation, exchange, 1 to 1 correspondence

Suggested Grade Level: 1, 2

Materials needed: Tongue depressors, glue, beans, small containers

Procedure: Put 10 drops of glue evenly spaced on a tongue depressor. Put beans on each drop of glue. The beanstick represents a set of 10. Give each child 10 tongue depressors, a container of beans, glue, and a plastic pill container. Allow child to discover how many sets of 10 he has? How many ones? Extra beans are placed in the pill bottle as ones.



Beanstick

1-9 Set Comparisons. Purpose: Equal and equivalent sets, commutative and associative properties

Suggested Grade Level: 3, 4

Materials needed: Colored rods or blocks

Procedure: Form a set with three different rods. Have children form an equal set. i.e. $[w, g, r,] = [g, r, w]$ The order in which the rods are placed does not matter. To be equal, the two sets must have exactly the same kinds of components. Equivalent sets $[w, g, y, k]$ equivalent, not equal to $[r, p, d, n]$ This activity should be repeated many times, varying the number of rods, etc.

1-10 Belonging. Purpose: Identifying sets

Suggested Grade Level: 1, 2, 3

Materials needed: None

Procedure: Have children take turns dividing the members of the class into sets. Let each child decide the criteria he wishes to use - sex, color of eyes, hair, shoes, etc. Suggest an empty set, e.g., pupils with orange eyes.

1-11 Heaps and Heaps. Purpose: inequalities, equalities, 1 to 1 correspondence, subtraction, counting to 100, division readiness

Suggested Grade Level: 1, 2

Materials needed: Counters

Procedure: With a friend, get a bag of counters. Put them into two unequal heaps. How many more counters are in the bigger heap? How many do you have altogether? Invite another friend to join you. Share the counters as equally as possible. Are there any left over?

1-12 What Do You See? Purpose: Estimation, counting, inequalities, set classification, ordinal numbers, subtraction, 1 to 1 correspondence

Suggested Grade Level: 1, 2

Materials needed: Multi-colored cubes, blocks, beads, or toys

Procedure: Have children name the colors they see. How many different colors are in the box? Without counting, guess which color is used on the greater number of blocks? Which color is used on the second greatest number of blocks? Which color is used on the third greatest number of blocks? Record the estimates. Separate blocks into color sets. Count the cubes in each set. Compare the count with your guess. Which was less, your guess or your count? How much less was it?

1-13 Word Sets. Purpose: Math vocabulary

Suggested Grade Level: 3-6

Materials needed: None

Procedure: Have a contest to see which child can find the most words beginning with "cent" and having some connection with 100, i.e., century, centenarian. Use the same procedure for the following word stems: geo, poly, uni, semi, bi, tri, dec. Display the results.

unit	geometry	bicycle	polygon
united	geology	binary	polynomial
unicorn	geography	bilateral	polytechnic
uniform	geopolitics	binomial	Polynesian

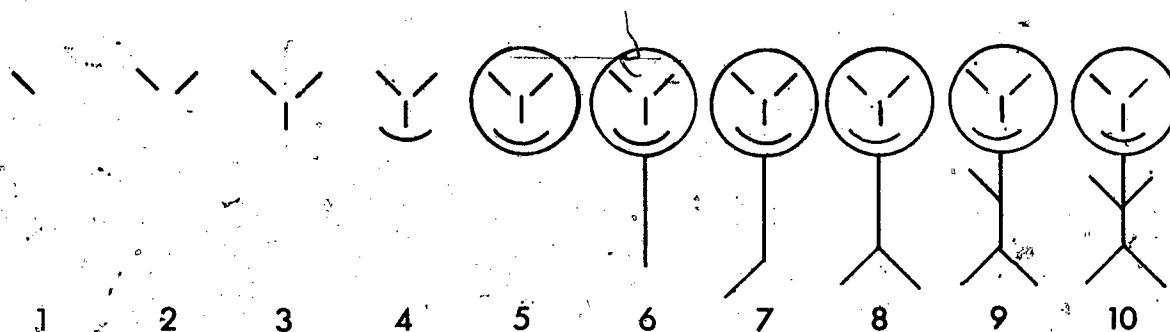
1-14 Wampum. Purpose: Estimation; tally counting, numeration, place value

Suggested Grade Level: 1-4

Materials needed: Assorted colored beads, laces, number track, containers

Procedure: Ask children for and record estimates of the total number of beads in a box. Discuss ways of keeping track such as actual counting: string beads in sets of 10, or string beads and knotting lace into a set of 10. Weigh a set of 10 beads - try then to find the number without actually counting. Lay 10 beads along a number track. When 10 beads are counted, put one bead in a bowl. When there are 10 beads in the bowl, remove them and place 1 bead in a box. Continue the process until all beads are counted.

Use tally strokes



1-15 Straw Polls. Purpose: Addition, subtraction, counting, sorting sets, equalities, inequalities, conservation

Suggested Grade Level: 1, 2

Materials needed: 20 plastic straws, some cut short, use two colors

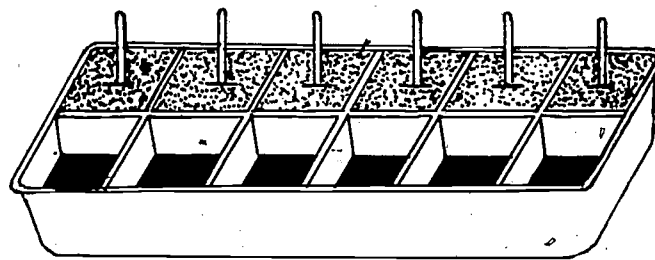
Procedure: Make different arrangements of shorts, longs, blues, reds. Ask children to count each variety. Have them draw each set and tell what is the same and what is different.

1-16 Crayon Candles. Purpose: Addition, subtraction, ordinal numbers, recording, counting

Suggested Grade Level: 1, 2

Materials needed: Box of crayons, egg carton, clay

Procedure: Stand crayons on one row of an egg box, attaching them with small pieces of clay. Ask which is first, second, third, etc. How many crayons? How many colors? What color is the third crayon? Take some away and ask which five are missing? Which three are missing? How many are missing? Make sums with two crayons and three crayons, etc. (This can also be done with pegs and a pegboard.)



1-17 Straw Men. Purpose: Addition, subtraction, counting, sets, geometric shape identification, inequalities

Suggested Grade Level: 1, 2

Materials needed: Colored straws, some long, some short, two colors

Procedure: Have children make geometric shapes with straws. Ask: How many reds did you use? How many blues did you use? How many longs did you use? How many shorts did you use? How many long red? How many long blue? How many short red? How many short blue? How many straws altogether did you use? Take away or add two red straws. How many straws are there now? Is this more or less than you started with? Have children name or describe the shapes they make.

1-18 Button, Button #1. Purpose: Cardinal numbers, counting, estimation, place value, time, sets, pictorial representation, measurement

Suggested Grade Level: 1, 2

Materials needed: Buttons, calendar

Procedure: Have children collect buttons. Look for a variety of colors, shapes, holes. On the class calendar, mark the date when the class will begin sorting the sets of buttons.

Activity: Let's guess how many buttons are in the container. Write it down. Count them. Record it. Let's describe the different kinds of buttons. Let's guess how many different kinds there are. Draw pictures of the different kinds of buttons. Separate them. Check how close you came to your guess. Was your guess too high, too low, or was it just right?

1-19 Button, Button #2. Purpose: Cardinal numbers, counting, estimation, subtraction, equalities and inequalities, sets, vocabulary, most, least, next, under

Suggested Grade Level: 1, 2

Materials needed: Buttons, board, tape

Procedure: Tape a red, blue, yellow, purple, and green button on a board. Have children name and write the color beneath the corresponding button. Arrange children in five groups. Ask each group to guess how many buttons of each color there are. Write the estimate under the color name on board. Now ask each group to count the exact number. Write the numeral under each guess. Ask which has most - least - next to most - next to least, etc. Compare the estimate to the actual count.

1-20 Button, Button #3. Purpose: Counting, shapes, sets, ordinal numbers

Suggested Grade Level: 1, 2

Materials needed: Buttons, board or tray

Procedure: On a board, tape one set of buttons to represent the following attributes: (a) number of holes - 0, 2, 4; (b) size - small, medium, large; (c) shape - round, square, other. Discuss these characteristics. Ask the children to find the buttons in collection with (a) no holes (b) two holes; (c) four holes; (d) smallest; (e) round but no holes. Ask them to count each group. Ask them to arrange them in order: first second, third, fourth, fifth.

1-21 Button, Button #4. Purpose: Counting, sets, addition, subtraction, recording, inequalities

Suggested Grade Level: K, 1, 2

Materials needed: Buttons

Procedure: Have children line buttons in rows by color. Ask how many more red buttons are there than purple ones. How many more blue buttons than green ones? How many more yellow buttons than red ones? How many more green buttons than purple ones? How many more red buttons than blue ones? How many more yellow buttons than purple ones? How many more blue buttons than purple ones? Count all reds and yellows. Record the count. Count greens and blues. How many purples and reds? Which color is used most? least?

1-22 Button, Button #5. Purpose: Counting, adding, subtracting, multiplying

Suggested Grade Level: 1, 2

Materials needed: Buttons, cards, unifix cubes, if available, can be used for these button activities

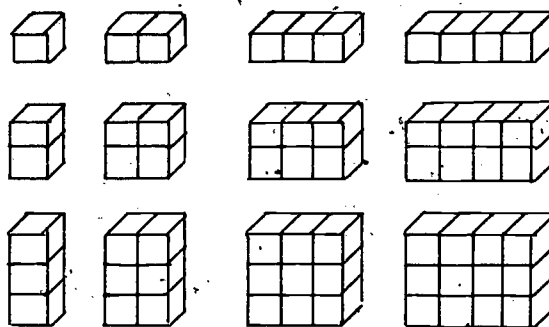
Procedure: Paste or sew buttons of two color varieties in rows on oak tag cards. For example to illustrate number facts for 12, make four rows of three buttons each. Make two rows of each color. Other arrays for the "12 facts" can be made and compared with this one. Directions: How many buttons on a card? Give equations and allow children to use buttons on card as counters.

1-23 Block Busters. Purpose: Pattern recognition, graphing, readiness, cardinal and ordinal numbering, whole number operations

Suggested Grade Level: 2-3

Materials needed: Cubical blocks

Procedure: Place blocks in rows and columns as shown below. Elicit properties such as the pile in the 2d row and the fourth column has eight blocks; it is "2 high" because that is true, of all the piles in the 2d row; and "4 long" because that is true of all the piles in the fourth column.



1-24 Patterns. Purpose: Odd and even numbers, addition, subtraction, ability to recognize 1, 2, 3, and 4 at a physical level without counting

Suggested Grade Level: 1, 2

Materials needed: Stern or unifix materials

Procedure: Discuss patterns that can help decide whether a number is odd or even. Ask children to put cubes of two colors on pattern blocks to show various combinations for numbers from 1-10.

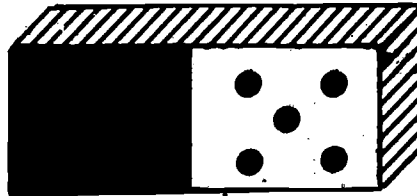
1-25 Domino Activities. Purpose: Sets, counting, recording, comparisons, addition, subtraction, missing addend, patterns-zero to six

Suggested Grade Level: 1, 2

Materials needed: Dominoes

Procedure: Ask child to put dominoes into sets. How many sixes, fives, fours, etc., are there? Which number has the most dots? Draw that particular one. Do you see the one without dots? Can you draw that one? Have children draw different patterns. Ask children to find dominoes with designated amounts. Find domino with 11 dots. Draw it. six dots. Draw it. etc. State: This domino has four dots and this domino has three dots. Four dots + three dots = seven dots. Make rows of dominoes and ask children to make and solve the equations or write equations and have children draw the dominoes. What else can we do with dominoes?

Cover one-half of a domino. Ask. "How many dots are hidden if there are nine dots on this domino?"



1-26 Assorted Counter Activities. Purpose: Sets, equalities and inequalities, ordinal numbers, counting

Suggested Grade Level: 1, 2

Materials needed: Assorted counters or attribute blocks

Procedure: Have assorted counters which differ in size, shape, color, thickness, weight, material, etc. Ask children to sort the counters according to distinguishing characteristic. A string around a group can help to set it apart from the other counters. How many counters are blue? not blue? How many counters are round? not round? How many counters are blue and round? not blue and not round? How many counters are wooden? not wooden? etc.

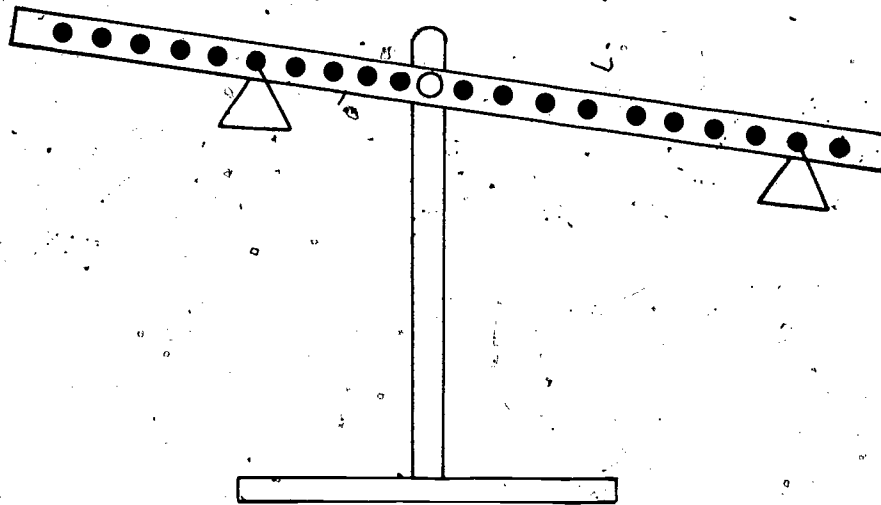
1-27 Up We Go. Purpose: Addition, subtraction, missing addends, left, right

Suggested Grade Level: 1, 2

Materials needed: Mathematical balance, unifix materials, set of examples for child to work on

Procedure: Discuss with the children the analogy of the balance fulcrum with the equal symbol in an equation. Have children correlate their equations with the balance. Further correlations may be made with equations represented by unifix materials.

$$\boxed{} \boxed{} \boxed{} \boxed{} \boxed{} + ? = \boxed{} \boxed{} \boxed{} \boxed{} \boxed{} \boxed{} \boxed{} \boxed{}$$



$$5 + \boxed{} = 9$$

On which arm of the balance should another weight be placed to make it level? Why? Where should it be put?

1-28 Pile Driver. Purpose: Whole number operations, equalities, inequalities

Suggested Grade Level: 1, 2

Materials needed: Set of congruent cubes or unifix blocks

Procedure: Put 16 cubes into four piles containing two, three, five, and six cubes. To "move," take one or more cubes from one pile and put them on another. Can you make all the piles the same height in two moves?

1-29 So Many. Purpose: Large numbers, multiplication, tally counting, estimation

Suggested Grade Level: 3, 4

Materials needed: Rulers, tape measures, trundle wheels, scales.

Procedure: Have children figure out ways to determine the number of bricks on wall, number of tiles on the floor, windows in all the classrooms, blades of grass in 2-square feet of lawn, number of hairs on one's head. They should realize the folly of merely counting each item.

1-30 Guessing Game. Purposes: Use of concrete models for comprehension of large numbers, sampling and estimation, measurement (volume, area), tallying

Suggested Grade Level: 3, 4

Materials needed: Large container with uniformly sized beads, scales, small containers, rulers, graph paper

Procedure: Ask children to estimate the number of beads in the box. Record answer on class chart. Now discuss ways to find out how many beads are really in the box if you may count some but not all the beads. Children should be free to explore many ways to solve this problem, i.e., count the number that will fill a pint container, use scales, count beads in a single layer, etc. Have children compare with each other their methods and results obtained. Ask a committee to make an accurate count. Tallying techniques will be helpful here.

1-31 Primes. Purpose: Prime numbers, multiplication and division facts

Suggested Grade Level: 3, 4, 5

Materials needed: Colored 1-inch cubes

Procedure: Arrange nine cubes to form a rectangular layer. How many different rectangles can you make? Repeat, varying the number of cubes used. How many ways can you do it with each number? Record results. For which numbers is there only one rectangular array? These are prime numbers. If you know the dimensions, could you figure out how many cubes are needed to make the rectangle? Do you have to do the arrangement to find the answer? If you knew how many cubes were used, could you find the dimensions?

1-32 Rectangular Arrays. Purpose: Prime and composite numbers

Suggested Grade Level: 3, 4, 5

Materials needed: Counters, i.e., cubes, colored discs, bottle caps, geoboards, pegboards

Procedure: Find all the numbers up to 100 that can be shown as a rectangular array with at least two columns. These are called composite numbers. Numbers which cannot be thus represented are prime numbers.

1-33 Rack Game. Purpose: Numeration, sequence, odd-even number

Suggested Grade Level: 3-6

Materials needed: Racko game or teacher-made version of same

Procedure: The game consists of a pack of 60 cards numbered 1-60 and four racks with 10 slots in each. Ten cards are dealt to each player. The player puts the cards in the rack from back to front as they are dealt. The object of the game is to have the cards go from low to high. The extra cards are the pack and players pick in turn. The card picked may be used to replace a card in the rack. Extra points are given for a 3, 4, 5 card sequence. Game may be varied by requiring use of only odd or only even numbers, or multiples of three, etc. Other variations: cards may be made using rational numbers, percents, directed numbers, geometric shapes whose area or perimeter is indicated.

1-34 1,000 to Nothing. Purpose: Place value, subtraction

Suggested Grade Level: 3, 4, 5

Materials needed: Dice (3)

Procedure: The game is played as follows: Players in turn roll one, two, or three dice. After the roll, dice are arranged to form a number. For example, roll of 5, 4, 1 can be 541, 451, 154, etc. This number is subtracted from 1000. Second player rolls and decides on number. This number is subtracted from the remaining difference. As the game proceeds, players must make many decisions - how many dice to use, what number, etc. Player whose roll gets the remainder to zero is the winner. Children should be encouraged to think of variations.

1-35 Different "Weights". Purpose: Numeration, whole number addition, subtraction, multiplication

Suggested Grade Level: 1, 2, 3

Material needed: Mathematical balance

Procedure: Assign two children to work together. Place a weight on any number, on one side of the balance. How many ways can the scale balance if you use one weight? two weights? three weights? Children should write the equations they discover on the balance.

1-36 Order. Purpose: Estimation, counting, addition, subtraction, color discrimination, cardinal and ordinal numbers

Suggested Grade Level: 3, 4

Material needed: Multicolored 1-inch cubes

Procedure: Without separating the colors, the child guesses which color is used for the greatest number of cubes, second greatest, etc. Record his estimate. The child then devises methods to verify his estimate. He compares his estimate with the actual count. How many cubes are in the entire set?

1-37 Bundling. Purpose: Number and numeration, place value, sets

Suggested Grade Level: 3

Materials needed: Have children prepare bundles of objects such as ice cream sticks. Use rubber bands to group sets of 10's and sets of 100's.

Procedure: Arrange children in two parallel rows or teams (whole class or small groups). Select a child to be the "tossor." Tossor stands between the rows and tosses sets onto the floor, i.e., three objects, two sets of 10, and four sets of 100 (423). The first person in each row is expected to write the numeral for the tossor's set. The first child who writes the number correctly scores a point for his team. If neither team responds correctly, the next child on each team tries. Participants move to the end of the line after they have tried to write the numeral. The game ends when each child has had a turn or when a predetermined score is reached.

1-38 Words, Words, Words. Purpose: Large numbers estimation, problem solving, graphing, averages, inequalities, mathematical vocabulary, data collection

Suggested Grade Level: 3 - 6

Materials needed: Newspaper or magazine page, marking pen, mathematical dictionary

Procedure: Ask pupils to guess the number of mathematical terms that appear on the page. Investigate a paragraph together, underline the words, and revise your estimates. Encourage different methods for attaining accurate estimates and actual counts. Outline various branches of mathematics and related topics that students should consider when they make their surveys.

Conduct a poll of your classmates or teachers to find how many mathematical words each child guesses are on the front page of today's newspaper. What was the average guess? Underline the mathematical terms and count them. Make a graph to compare the guesses with the actual number of terms you counted. How many words are used in a newspaper column? If the newspaper had 45 pages, how many words, approximately, would it have altogether?

1-39 Large Numbers. Purpose: Estimation, weight, practical application, problem solving, sampling

Suggested Grade Level: 5, 6

Materials needed: Scale, 1 pound box of rice, small containers

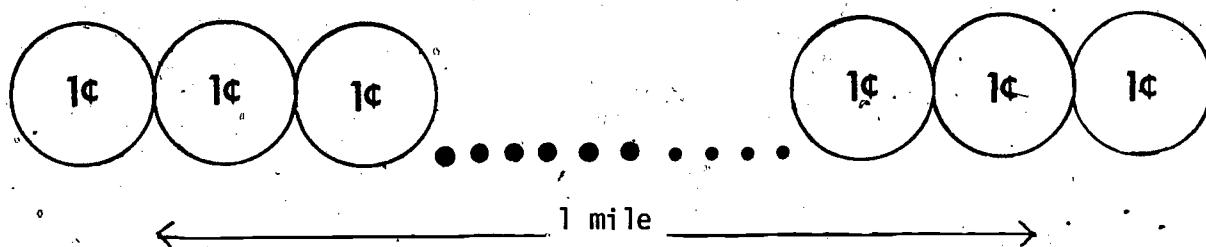
Procedure: Large numbers have always had a special fascination for children. Students need concrete models to develop an appreciation of the meaning of large numbers. The technique of sampling and extrapolating is used in industry and in research. A discussion of quality and quantity controls can be an outgrowth of the following problems: Without counting all of the grains, find how many grains of rice are in a 1-pound box. Figure out the weight of 1 million grains of rice. How much would it cost? Would it fit in your classroom?

1-40 Penny, Penny. Purpose: Numeration, large numbers, sampling for problem solving, weights, operations with fractions

Suggested Grade Level: 5, 6

Materials needed: Tape measure or ruler, scale, about 20 pennies

Procedure: If you line up a row of pennies for 1 mile, how many pennies would you use?



How much would all these pennies weigh? Estimate the weight of fractional parts of a mile of pennies.

1-41 Your Weight in Gold. Purpose: Numeration, estimating, multiplication, weighing, monetary values, graphing

Suggested Grade Level: 5, 6

Materials needed: Scale, graph paper

Procedure: Have students find their own weight and the weights of several friends. How much would each be worth if they were made of pennies? (3 pennies = 1 oz.) How much would each be worth if they were made of gold? Graph your results. Look up actual worth and repeat. (Assume 1 oz. = \$65)

1-42 Small Change. Purpose: Numeration, weighing, whole number operations

Suggested Grade Level: 5, 6

Materials needed: Pennies, nickels, quarters, scale

Procedure: Have students find how many pennies make 1 ounce. How much would \$5,000 in pennies weigh? Try this same experiment for nickels and quarters. Which would you rather have: 1 lb. of quarters; 2 lbs. of nickels; 3 lbs. of pennies? How could you tell whether your choice is the best?

1-43 From Here to Eternity. Purpose: Numeration, estimation, time measure, data recording, graphing

Suggested Grade Level: 5, 6

Materials needed: None

Procedure: How many is a million? Have you lived a million days? Have you lived a million seconds? How many days have you lived? How many seconds have you lived? Record your findings (graph, table, picture).

1-44 Intrinsic Values. Purpose: Research, numeration, factors

Suggested Grade Level: 5, 6

Materials needed: Library resources

Procedure: Have pupils find out about superstitions concerning numbers. Example: "Lucky" 7, "Bad Luck" 13. What are perfect numbers? amicable numbers? defective numbers? abundant numbers?

1-45 What Can You Do? Purpose: Numeration, large numbers, multiplication, time measurement, tallying

Suggested Grade Level: 3, 4

Material needed: Clock with second hand

Procedure: Using a clock with a second hand, see how many pencil strokes you can make in 1 minute, i.e., if you could work 8 hours a day, how long would it take you to make a million strokes?

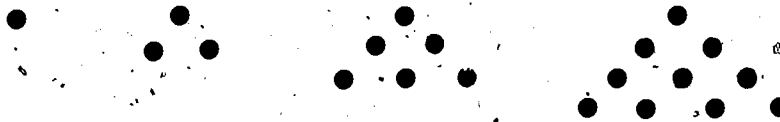
1-46 Figurative (Geometric) Numeration. Purpose: Numeration, patterns, Arithmetic, geometry, basic arithmetic operations

Suggested Grade Level: 5, 6

Materials needed: counters, Dienes AEM, regular polygons

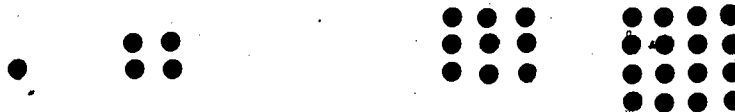
Procedure: The Pythagorean mathematicians were interested in interpreting arithmetic through geometry. They used symmetric arrays to represent numbers. They discovered properties of several series of numbers from these diagrams.

I. Triangular Numbers: 1, 3, 6, 10, ...



What is the next triangular number? the 10th; the 20th? What happens when you add or subtract adjacent triangular numbers? Write a description of triangular numbers.

II. Square Numbers: 1, 4, 9, 16, ...



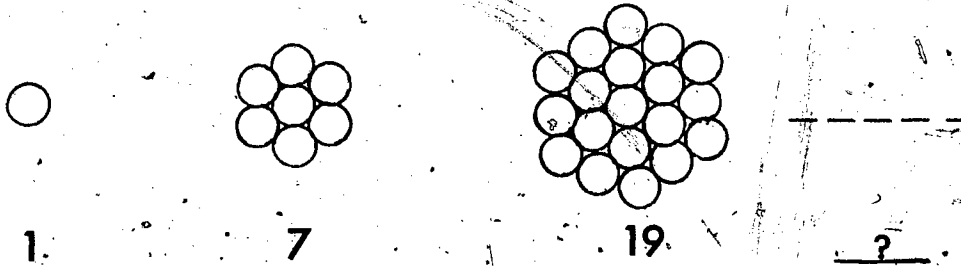
What is the next square number? the 10th? the 20th? How can you compare triangular numbers with square numbers? How are odd numbers related to square numbers? ($1 + 3 = 4$, $1 + 3 + 5 = 9$, ...) Write a description of square numbers.

III. Other figurative numerals: Encourage investigations of arrays and patterns for other polygonal numbers (pentagonal, hexagonal, rectangular). Instead of using dots in your arrays for figurative numbers, substitute a set of congruent regular polygons. What do you see happening?

Example: For triangular numbers

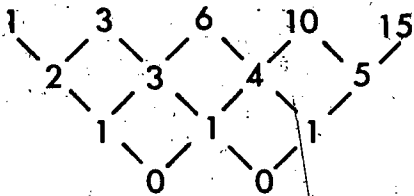


Can you find number patterns when you "pack" sets of congruent circles in rings so that the circles just touch each other?

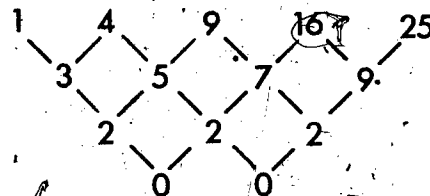


Consider patterns developed by taking successive differences:

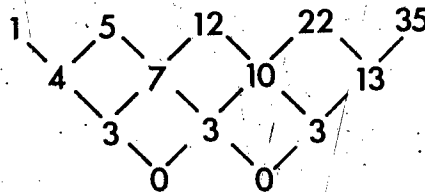
Triangular Numbers



Square Numbers



Pentagonal Numbers

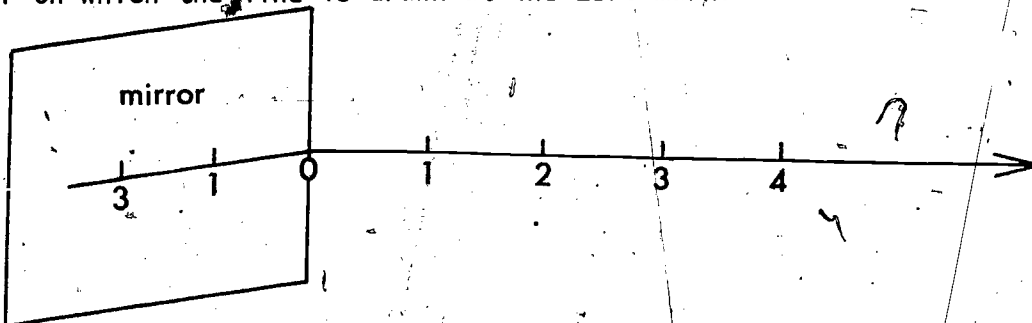


1-47 On and On. Purpose: Introduction to directed numbers, symmetry, angle measure

Suggested Grade Level: 5, 6

Materials needed: Number line, counters, mirror

Procedure: Draw a number line, starting with 0 and extending as far as you like. Place a mirror at right angles to the number line and the paper on which the line is drawn at the zero mark.



What image is in the mirror? Move a counter along the paper number line. What happens to its image in the mirror?

Categorical Listing

Selective Manipulative Materials for Math Lab Use

Number and Numeration: Counting and/or classification, place value

Improvised materials: Acorns, beans, bottle caps, buttons, classroom equipment (books, erasers, pencils, window panes, desks, chairs, floor tiles, etc.), corks, discs, fingers, foot and handprints, graph paper, horse chestnuts (conkers), ice cream sticks, leaves, money, number lines, pebbles, pine cones, sample swatches of various materials, seasonal paper cut-outs (pumpkins, snowmen, hearts) seeds, shells, straws, telephone directories, tiles, toothpicks, twigs

Improvised Games: Hopscotch, Buzz

Commercial materials: Abaci and counting frames, attribute blocks, beads, binary counter blocks, clay or plasticine, calendars, clocks and other timers, clothespins, cubes, dials, directed number slide rule, discs, dice, dominoes, felt shapes and flannel board, fundamath, geoblocks, geoboards, magnetic shapes, mechanical and/or electronic desk calculators, metal washers, meters, number lines, playing cards, popit beads, small toys (animals, vehicles, dolls, marbles, checkers, etc.) slide rules, squared materials, structural materials, centimeter rods, tern material, unifix rods, multibase arithmetic blocks, stencil graph, tally registers, tongue depressors, trundle wheels.

Commercial Games: Arithmecubes, Lego, Kalah, Oh-Wah-Lee, Chutes and Ladders, Bingo, Parchesi, Back-Up Three, Racko, Ranko, Rook, Yahtzee, and many more

General Supplies:

- Paper - ruled, unruled
- graph - squares (1/10", 1/4", 1/2", 1"), isometric
- gummed shapes
- construction, art, newsprint, tracing
- brown wrapping, wallpaper, carbon
- library cards - 3" x 5", 5" x 8"
- corrugated cardboard

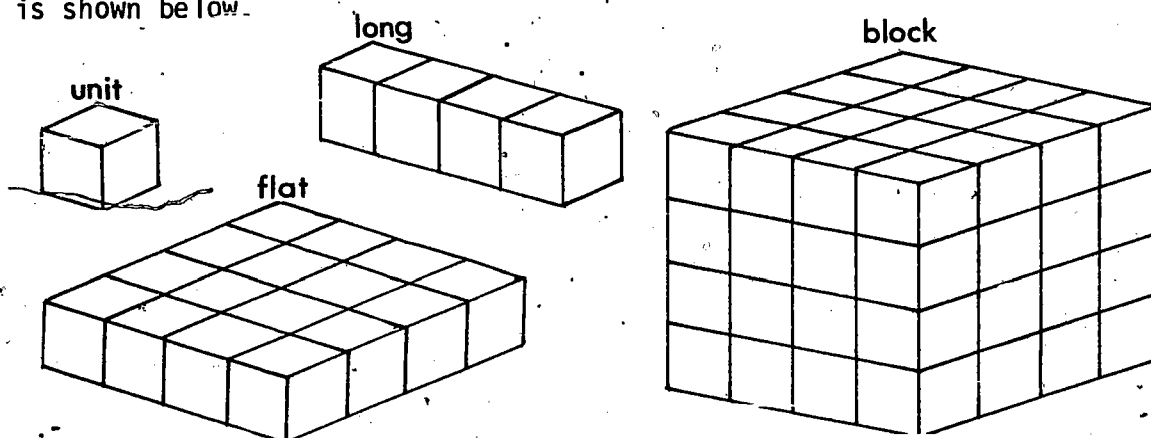
Thumb tacks, paper fasteners, clips, pins, cellophane and masking tape, glue, paste, scissors, strings, rubber bands, sponges, laces, yarn, straw, pipe cleaners, plasticine or clay, pencils, paints, brushes, crayons, stapler, stamp pads, filing folders, tool chest, balsa wood, screws, nails, styrofoam forms

Storage Containers: 1) Cardboard boxes, rectangular and cylindrical (covered with vinyl wallpaper for strength and color); 2) baskets, crates, and bushels from the produce markets; 3) wire hangers and clothespins or clamps for display and paper storage; 4) emptied aluminum, tin, cardboard, plastic food containers - checked for sharp edges and then painted; 5) commercial containers

BLOCKS - 1) Attribute (logic) Blocks - Wooden or plastic or rubber shapes that provide opportunities for classification according to size, color, and other distinguishing characteristics. Various activities can be designed, using these materials, to promote understanding of set operations.

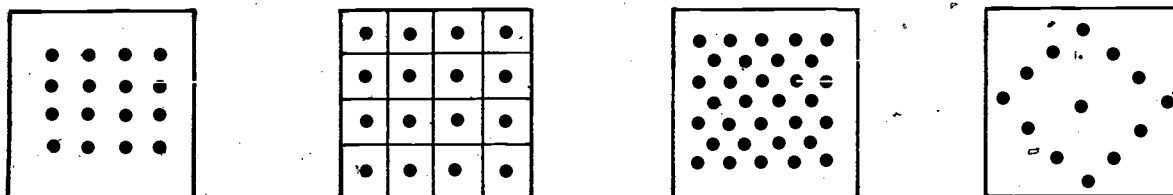
2) Geoblocks - Poleidoblocs, Ornabo, Discovery Blocks, Polyhedron models) - Various shapes can provide opportunities for exploring properties of solid geometric figures. They extend to three dimensions concepts explored with pattern blocks. Pupils enjoy assembling equivalent sets and playing identification guessing games with this material.

3) Multibase Arithmetic Blocks (Dienes or Tilloch Blocks) - These materials provide experiences in the decimal and other number systems. Place value is developed through a three dimensional geometric interpretation. The unit is a cubic centimeter of hardwood. The other blocks are multiples of the base unit. For example, the set for base 4 is shown below.



INVICTA #100 BALANCE AND EQUALIZER - One face of the long-armed beam of these balances is numbered 1-10 outward from the fulcrum. The reverse face is blank. The balance has pegs, while the equalizer has slots for holding easily manipulated metal weights. Arithmetic operations with whole numbers and fractions, field properties, and equations may be explored with this equipment.

GEOBOARDS - These are generally square boards of wood or plastic with varied lattice patterns made by pegs or nails. Children stretch rubber bands around the pegs or nails to produce geometric shapes. Geoboards may be used for learning, arithmetic operations, set concepts, numeration, and geometric relations.



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